9 Suppose m is a positive integer and $p_0, p_1, \ldots, p_m \in \mathcal{P}(\mathbf{F})$ are such that each p_k has degree k. Prove that $p_0, p_1, \ldots, p_m$ is a basis of $\mathcal{P}_m(\mathbf{F})$.

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Suppose that $P_0, \ldots, P_m \in \mathcal{P}(\mathbb{F})$ are such that P_k has degree k.

Since P_0 is a constant and $P_0 \neq 0$.

Then, $1 = P_0 / P_0$

We can write $P_1 = ax+b$ forsome $a, b \in \mathbb{F}$ with $a \neq 0$.

$$x = \frac{(P_1 - b)}{a}$$

We can write $P_2 = \bar{a}x^2 + \bar{b}x + \bar{c}$

$$x^2 = \frac{1}{(\bar{a})}\left[P_2 - \bar{c} - \bar{b}\left(\frac{P_1 - b}{a}\right)\right]$$

Continue this process x^{i} as a linear combination of $p_0, \ldots, p_i$ for $i=1,2,\ldots,k$

Thus $p_0, \ldots, p_k$ spans $\mathcal{P}_m(\mathbb{F})$ and

~~So $p_0, \ldots, p_k$ i~~

We know that $\dim(\mathcal{P}_k(\mathbb{F})) = k+1$

By 2.42, $p_0, \ldots, p_k$ is a basis for $\mathcal{P}_k(\mathbb{F})$.